

# DIFFERENCE BETWEEN DENAVIT - HARTENBERG (D-H) CLASSICAL AND MODIFIED CONVENTIONS FOR FORWARD KINEMATICS OF ROBOTS WITH CASE STUDY

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**Abstract:** This paper highlights the difference between the D-H classical convention and D-H modified convention for RR manipulator. The forward kinematics ends at a frame, whose origin lies on the last joint axis  $z_2$  as per modified D-H convention,  $a_2$  does not appear in the link parameters whereas  $a_2$  appears in the link parameters of D-H classical convention.

**Keywords:** RR manipulator, D-H classical convention, D-H modified convention

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## 1. KINEMATIC CHAIN

A robotic manipulator may be considered as set of links connected in a chain called **kinematic chain** by joints (figure 1). The simple joints are prismatic joint (figure 2a) and revolute joint (figure 2b). The prismatic joints permit a linear motion and the revolute joints allow a rotary motion. The revolute and prismatic joints exhibit one degree of freedom (dof).

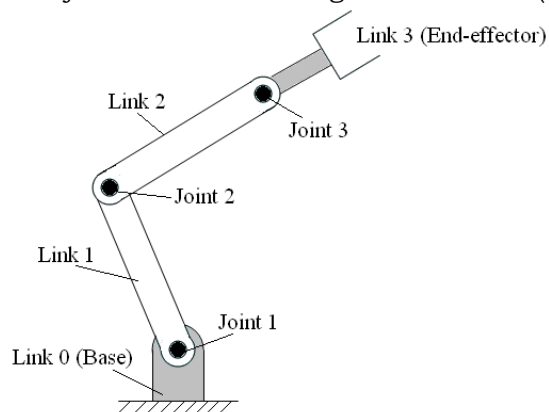


Figure 1: A three-degrees of freedom robotic manipulator

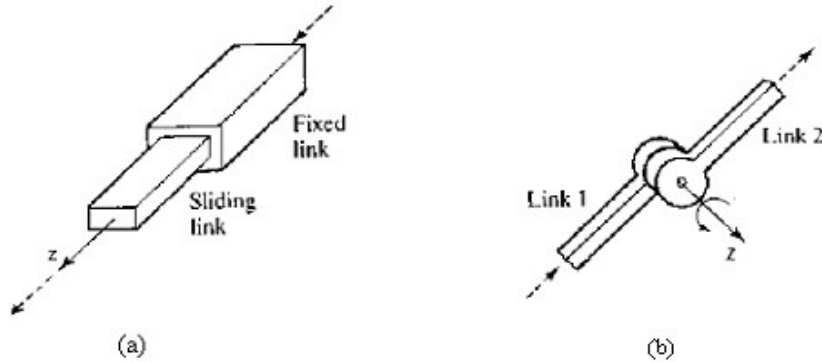


Figure 2: Joints (a) Prismatic joint, and (b) revolute joint

Typical robots are *serial-link manipulators* comprising a set of bodies, called *links*, in a chain, connected by *joints*<sup>1</sup>. In this book, each joint has one degree of freedom, either translational or rotational. Note that the assumption does not involve any real loss of generality, since joints such as a ball and socket joint (two degrees-of-freedom) or a spherical wrist (three degrees-of-freedom) can always be thought of as a succession of single degree-of-freedom joints with links of length zero in between. For a manipulator with  $n$  joints numbered from 1 to  $n$ , there are  $n+1$  links, numbered from 0 to  $n$ . Link 0 is the base of the manipulator, generally fixed, and link  $n$  carries the end-effector. Joint  $i$  connects links  $i$  and  $i-1$ . when joint  $i$  is acutated, link  $i$  moves. A link can be specified by two numbers, the *link length* and *link twist*, which define the relative location of the two axes in space. Joints may be described by two parameters. The *offset length* is the distance from one link to the next along the axis of the joint. The *joint angle* is the rotation of one link with respect to the next about the joint axis.

A coordinate frame is attached rigidly to each link. To facilitate describing the location of each link we affix a coordinate frame to it: frame  $i$  is attached to link  $i$ . When the robotic manipulator executes a motion, the coordinates of each point on the link are constant. Each joint has a joint axis with respect to which the motion of joint is described. By convention, the  $z$ -axis of a coordinate frame is aligned with the joint axis.

Description of an end-effector in space requires a minimum of six degrees of freedom. Typical robotic manipulators have five or six degrees of freedom. The objective of forward kinematic analysis is to determine the cumulative effect of the entire set of joint variables.

The displacement of joint is denoted by  $q_i$  and is called joint variable. The collection of joint variables

$$q = [q_1, q_2, \dots, q_n]^T \quad (1)$$

is called the joint vector.

The position of the end-effector is denoted by the dimensional vector

$$r = [r_1, r_2, \dots, r_m]^T \quad (2)$$

The relation between  $r$  and  $q$  determined by the manipulator mechanism is given by

$$r = f(q) \quad (3)$$

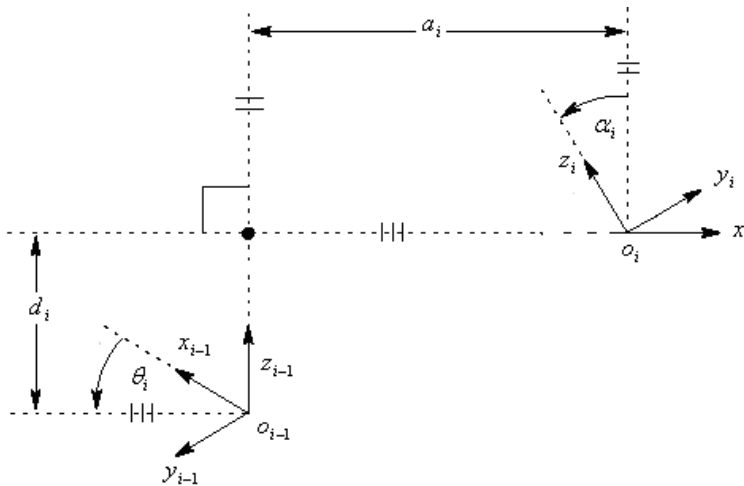


Figure 3: Description of link parameters

## 2. LINK AND JOINT PARAMETERS

Links are the solid bits between joints. Links have a *proximal* end closest to the base and a *distal* end closest to the tool. The proximal end of the link has the lower joint number. Each type of link has 4 parameters, 2 directions of translation and 2 axes of rotation. These are called the *link parameters*.

Let us consider a binary link of an articulated mechanism as shown in figure 3. It establishes a rigid connection between two successive joints numbered  $i$  and  $(i+1)$ . Its geometry in terms of size and shape can be described very simply in terms of only two parameters:

1. The distance  $a_i$ , measured along the common normal to both axes. The variable  $a_i$  is called the link length;
2. The twist angle  $\alpha_i$ , defined as the angle between both joint axes. The variable  $\alpha_i$  is called the twist angle. The twist angle is measured between the orthogonal projections of joint axes  $i$  and  $(i+1)$  onto a plane normal to the common normal.

If the relative motion is restrained to joints of revolute, and prismatic, the relative displacement occurring at joint  $i$  may also be described in terms of two parameters:

1. The rotation  $\theta_i$  about the joint axis. The variable  $\theta_i$  is called joint angle;
2. The displacement  $d_i$  along the same axis. The variable  $d_i$  is called link offset.

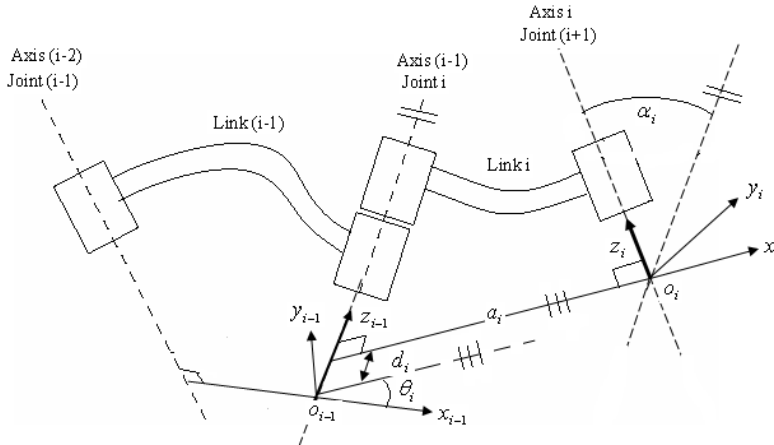


Figure 4: Description of link and joint parameters

## 2.1 Link Parameters

A link  $i$  is connected to two other links (i.e. link  $i - 1$  and link  $i + 1$ ). Thus two joint axes are established at both ends of the link as shown in figure 4. Joints  $i-1$  and  $i$  are connected by link  $i-1$ . Joints  $i$  and  $i+1$  are connected by link  $i$ . The significance of links is that they maintain a fixed configuration between the joints which can be characterized by two parameters  $a_i$  and  $\alpha_i$ , which determine the structure of the link. They are defined as follows:

1.  $a_i$  is the shortest distance measured along  $x_i$  axis from the point of intersection of  $x_i$  axis with  $z_{i-1}$  axis to the origin.
2.  $\alpha_i$  is the angle between the joint axes  $z_{i-1}$  and  $z_i$  axes measured about  $x_i$  axis in the right hand sense.

## 2.2 Joint Parameters

A joint axis establishes the connection between two links. This joint axis will have two normals connected to it, one for each link. The relative position of two such connected links  $i-1$  and  $i$  is given by  $d_i$  which is the distance measured along the joint axis  $z_{i-1}$  between the common normals. The joint angle  $\theta_i$  between the common normals is measured in a plane normal to the joint axis  $z_{i-1}$ . Hence, the parameters  $d_i$  and  $\theta_i$  are called

distance and angle between adjacent links. They determine the relative position of neighboring links. For revolute joint,  $\theta_i$  varies and  $d_i$  is a fixed length (i.e., zero or constant). For prismatic joint,  $d_i$  varies and  $\theta_i$  is zero or constant.

### 3. LINK FRAMES

Let us define a coordinate frame  $o_i x_i y_i z_i$  attached to link  $i$  as follows:

1.  $z$  - axis is along the rotation direction for revolute joints, along the translation direction for prismatic joints.
2. The  $z_{i-1}$  axis lies along the axis of motion of the  $i$ th joint.
3. The origin  $o_i$  is located at the intersection of joint axis  $z_i$  with the common normal to  $z_i$  and  $z_{i-1}$ .
4. The  $x_i$  axis is taken along the common normal and points from joint  $i$  to joint  $i+1$ .
5. The  $y_i$  axis is selected to complete right-hand frame. The  $y_i$  axis is defined by the cross product  $y_i = z_i \times x_i$ .

Showing only  $z$  and  $x$  axes is sufficient, drawing is made clearer by **NOT** showing  $y$  axis.

By the above procedure, the link frames for links 1 through  $n-1$  are determined.

### 4. DENAVIT - HARTENBERG (D-H) CONVENTION

A commonly used convention for selecting frames of reference in robotic application is Denavit-Hartenberg convention. In this convention, the position and orientation of the end-effector is given by

$$H = {}^0T_n = {}^0T_1 {}^1T_2 {}^2T_3 \dots {}^{n-1}T_n \quad (4)$$

where,

$${}^{i-1}T_i = \begin{bmatrix} {}^iR_{i-1} & {}^i d_{i-1} \\ 0 & 1 \end{bmatrix} \quad (5)$$

Many people are not aware that there are two quite different forms of Denavit-Hartenberg representation for the kinematics of serial-link manipulators:

1. Classical convention as per the original paper of Denavit and Hartenberg [1], and used in textbooks such as by Paul [2], Fu et. al [3], or Spong et.al [4].
2. Modified convention as introduced by John J. Craig in his textbook [5] and Tsuneo Yoshikawa in his textbook [6].

Both notations represent a joint as 2 translations ( $a$  and  $d$ ) and 2 angles ( $a$  and  $\theta$ ). However the expressions for the link transform matrices are quite

different. In short, you must know which kinematic convention your Denavit-Hartenberg parameters conform to. Unfortunately many sources in the literature do not specify this crucial piece of information, perhaps because the authors assume everybody uses the particular convention that they do.

#### 4.1 Classical Convention

The link and joint parameters in the classical convention as shown in figure 5 are as follows:

- Link length,  $a_i$  is the offset distance from  $o_i$  to the intersection of the  $z_{i-1}$  and  $x_i$  axes along the  $x_i$  axis;
- Twist angle,  $\alpha_i$  is the angle from the  $z_{i-1}$  axis to the  $z_i$  axis about the  $x_i$  axis;
- Offset length,  $d_i$  is the distance from the origin of the  $(i-1)$  frame to the intersection of the  $z_{i-1}$  axis with the  $x_i$  axis along the  $z_{i-1}$  axis;
- Joint angle,  $\theta_i$  is the angle between the  $x_{i-1}$  and  $x_i$  axes about the  $z_{i-1}$  axis.

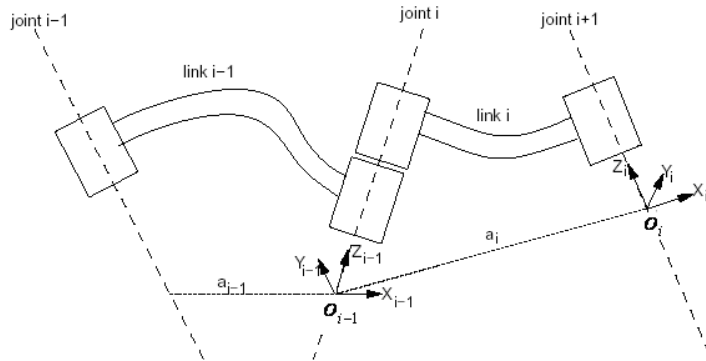


Figure 5: Classical convention

The positive sense of  $\alpha_i$  and  $\theta_i$  is shown in figure 6.

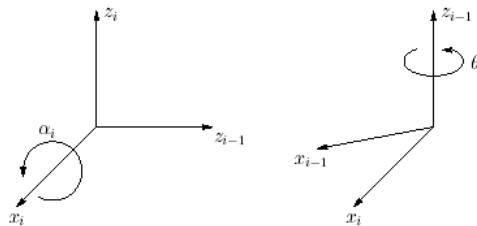


Figure 6: Positive sense of  $\alpha_i$  and  $\theta_i$

The D-H parameters are tabulated in table 1.

Table 1: D-H parameters for classical convention

Link, $i$	$\alpha_i$	$a_i$	$d_i$	$\theta_i$
1				
2				

The frame transformation  ${}^{i-1}T_i$  describing the finite motion from link  $i-1$  to link  $i$  may then be expressed as the following sequence of elementary transformations, starting from link  $(i-1)$ :

1. A rotation  $\theta_i$  about  $z_{i-1}$  axis;
2. A translation  $d_i$  along the  $z_{i-1}$  axis ;
3. A translation  $a_i$  along the  $x_i$  axis
4. A rotation  $\alpha_i$  about  $x_i$  axis.

The homogeneous transformation  ${}^{i-1}T_i$  is represented as a product of four basic transformations as follows:

$$\begin{aligned}
 {}^{i-1}T_i &= R(z_{i-1}, \theta_i) T(z_{i-1}, d_i) T(x_i, a_i) R(x_i, \alpha_i) \\
 &= \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)
 \end{aligned}$$

**Algorithm for classical D-H Convention:**

**Step - 1:** Identify and number the links starting with base and ending with end-effector. The links are numbered from 0 to  $n$ . The base frame is  $\{0\}$  and the end-effector frame is  $\{n\}$ . Locate and label the joint axes  $z_0, \dots, z_{n-1}$ .

**Step - 2:** The location of frame to the base is arbitrary. The  $x_0$  axis, which is perpendicular to  $z_0$ , is chosen to be parallel to  $x_1$  when the first joint angle variable  $\theta_1 = 0$  in the home position. The  $y_0$  axis is defined by the cross product  $y_0 = z_0 \times x_0$ .

For  $i = 1, \dots, n$ , perform Steps 3 to 5.

**Step - 3:** Locate the origin  $o_i$  where the common normal to  $z_i$  and  $z_{i-1}$  axes intersect at  $z_i$  axis. If  $z_i$  axis intersects  $z_{i-1}$  axis, locate the origin  $o_i$  at this point of intersection. If  $z_i$  and  $z_{i-1}$  axes are parallel, locate the origin  $o_i$  in any convenient position along  $z_i$  axis.

**Step - 4:** Establish  $x_i$  along the common normal between  $z_i$  and  $z_{i-1}$  through  $o_i$ . The  $x_i$  axis is fixed perpendicular to both  $z_i$  and  $z_{i-1}$  axes and points away from  $z_i$  axis. The origin of frame  $\{i\}$  is at the intersection of  $z_i$  and  $x_i$  axes.

- If the z-axes of two successive joints are intersecting, there is no common normal between them (or it has zero length). We will assign the x-axis along a line perpendicular to the plane formed by the two axes. If  $z_i$  and  $z_{i-1}$  axes intersect, choose the origin at the point of intersection. The  $x_i$  axis will be perpendicular to the plane containing  $z_i$  and  $z_{i-1}$ . In this case, the parameter  $a_i$  equals 0.
- If two joint z-axes are parallel, there are an infinite number of common normals present. We will pick the common normal that is collinear with the common normal of the previous joint. a common method for choosing  $o_i$  is to choose the normal that passes through  $o_{i-1}$  as the  $x_i$  axis;  $o_i$  is then the point at which this normal intersects  $z_i$ . In this case,  $d_i$  is equal to zero. Since the  $z_i$  and  $z_{i-1}$  axes are parallel,  $\alpha_i$  is equal to zero.
- If  $z_i$  and  $z_{i-1}$  axes coincide, the origin lies on the common axis. If the joint  $i$  is revolute, the origin is located to coincide with origin of frame ( $i$ ) and  $x_i$  axis coincides with  $x_{i-1}$  axis. If the joint  $i$  is prismatic,  $x_i$  axis is chosen parallel to  $x_{i-1}$  axis and the origin is located at the distal end of the link  $i$ .

**Step - 5:** The  $y_i$  axis is selected to complete right-hand frame. The  $y_i$  axis is defined by the cross product  $y_i = z_i \times x_i$ .

**Step - 6:** Establish the end-effector frame ( $o_n$ ) as shown in figure 7. assuming the nth joint is revolute, set  $z_n = a$  (approach direction) along the direction  $z_{n-1}$  and pointing away from the link  $n$ . Establish the origin  $o_n$  conveniently along  $z_n$ , preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set  $y_n = s$  in the sliding direction along which the fingers the gripper slide to open or close and set  $x_n = n$  as  $s \times a$ . If the tool is not a simple gripper set  $x_n$  and  $y_n$  conveniently to form a right-hand frame.

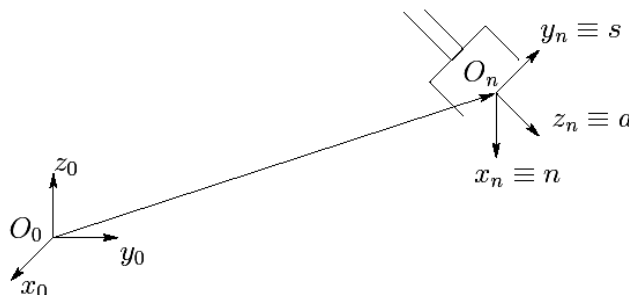


Figure 7: Establishing frame for end-effector

**Step - 7:** Create a table 2 of link parameters  $a_i$ ,  $\alpha_i$ ,  $d_i$ , and  $\theta_i$ .



Table 2: Link parameters

Link, $i$	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1				
2				

- The link length  $a_i$  is the shortest distance between  $z_{i-1}$  and  $z_i$  axes. It is measured as the distance along the direction of  $x_i$  from the intersection of  $z_{i-1}$  and  $x_i$  to the origin of the  $i$ th coordinate frame. For intersecting joint axes the value of  $a_i$  is zero. It has no meaning for prismatic joints and is set to zero in this case.
- The offset angle,  $\alpha_i$ , is measured from  $z_{i-1}$  axis to  $z_i$  about the  $x_i$  axis, again using a right-hand rule. For most commercial manipulators the offset angles are multiples of  $90^\circ$ .
- The distance between links,  $d_i$ , is the distance from the  $x_{i-1}$  to the  $x_i$  axis measured along the  $z_{i-1}$  axis. If the joint is prismatic,  $d_i$  is the joint variable. In the case of a revolute joint, it is a constant or zero.
- $\theta_i$  is the angle from the  $x_{i-1}$  to the  $x_i$  axis measured about  $z_{i-1}$  axis. This is defined using a right-hand rule since both  $x_{i-1}$  and  $x_i$  are perpendicular to  $z_{i-1}$ . The direction of rotation is positive if the cross product of  $x_{i-1}$  and  $x_i$  defines the  $z_{i-1}$  axis.  $\theta_i$  is the joint variable if the joint  $i$  is revolute. In the case of a prismatic joint it is a constant or zero.

**Step - 8:** Form the homogeneous transformation matrices  ${}^{i-1}T_i$  by substituting the above parameters into equation (6).

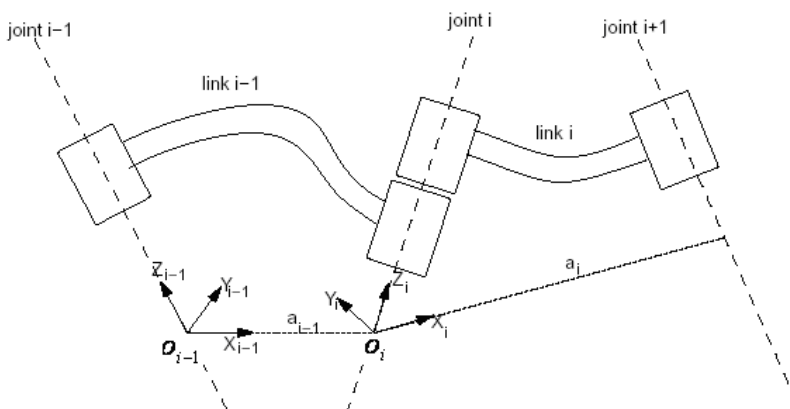


Figure 8: Modified convention

**Step - 9:** Form  ${}^0T_n = {}^0T_1 {}^1T_2 {}^2T_3 \dots {}^{n-1}T_n$ . This gives the position and orientation of the end-effector frame expressed in the base coordinates.

**Note:** The origin of the base frame is coincident with the origin of the joint 1. This assumes that the axis the first joint is normal to the  $xy$  plane.

### 4.2 Modified Convention

The link and joint parameters in the classical convention as shown in figure 8 are as follows:

- Twist angle,  $\alpha_{i-1}$  is the angle between  $z_{i-1}$  to  $z_i$  measured about  $x_{i-1}$
- Link length,  $a_{i-1}$  is the distance from  $z_{i-1}$  to  $z_i$  measured along  $x_{i-1}$
- Offset length,  $d_i$  is the distance from  $x_{i-1}$  to  $x_i$  measured along  $z_i$
- Joint angle,  $\theta_i$  is the angle between  $x_{i-1}$  to  $x_i$  measured about  $z_i$

The D-H parameters are determined as per table 3.

Table 3: D-H parameters for modified convention

Link, $i$	$\alpha_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1				
2				

The frame transformation  ${}^{i-1}T_i$  describing the finite motion from link  $i-1$  to link  $i$  may then be expressed as the following sequence of elementary transformations, starting from link  $(i-1)$ :

1. A rotation  $\alpha_{i-1}$  about  $x_{i-1}$ .
2. A translation  $a_{i-1}$  along the  $x_{i-1}$  axis
3. A rotation  $\theta_i$  about  $z_i$ ;
4. A translation  $d_i$  along the same axis  $z_i$ ;

The homogeneous transformation  ${}^{i-1}T_i$  is represented as a product of four basic transformations as follows:

$$\begin{aligned}
 {}^{i-1}T_i &= R(x_{i-1}, \alpha_{i-1}) T(x_{i-1}, a_{i-1}) R(z_i, \theta_i) T(z_i, d_i) \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_{i-1} & -S\alpha_{i-1} & 0 \\ 0 & S\alpha_{i-1} & C\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -d_i S\alpha_{i-1} \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & d_i C\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)
 \end{aligned}$$

An alternative representation of  ${}^{base}T_{end-effector}$  can be written as

$${}^{base}T_{end-effector} = {}^b T_e = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

where  $r_{kj}$ 's represent the rotational elements of transformation matrix ( $k$  and  $j = 1, 2$  and  $3$ ).  $p_x$ ,  $p_y$  and  $p_z$  denote the elements of the position vector. For a six jointed manipulator, the position and orientation of the end-effector with respect to the base is given by

$${}^0T_6 = {}^0T_1(q_1) {}^1T_2(q_2) {}^2T_3(q_3) {}^3T_4(q_4) {}^4T_5(q_{51}) {}^5T_6(q_6) \quad (9)$$

where  $q_i$  is the joint variable (revolute or prismatic joint) for joint  $i$ , ( $i = 1, 2, \dots, 6$ ).

**Algorithm for modified D-H Convention:**

**Step - 1:** Assigning of base frame: the base frame  $\{0\}$  is assigned to link 0.

The base frame  $\{0\}$  is arbitrary. For simplicity chose  $z_0$  along  $z_1$  axis when the first joint variable is zero. Using this convention, we have  $a_0 = 0$  and  $\alpha_0 = 0$ . This also ensures that  $d_1 = 0$  if the joint is revolute and  $\theta_1 = 0$  if the joint is prismatic.

**Step - 2:** Identify links. The link frames are named by number according to the link to which they are attached (i.e. frame  $\{i\}$  is attached rigidly to link  $i$ ). For example, the frame  $\{2\}$  is attached to link 2.

Identify joints. The  $z$ -axis of frame  $\{i\}$ , called  $z_i$ , is coincident with the joint axis  $i$ . The link  $i$  has two joint axes,  $z_i$  and  $z_{i+1}$ . The  $z_i$  axis is assigned to joint  $i$  and  $z_{i+1}$  is assigned to joint  $(i+1)$ .

For  $i = 1, \dots, n$  perform steps 3 to 6.

**Step - 3:** Identify the common normal between  $z_i$  and  $z_{i+1}$  axes, or point of intersection. The origin of frame  $\{i\}$  is located where the common normal ( $a_i$ ) meets the  $z_i$  axis.

**Step - 4:** Assign the  $z_i$  axis pointing along the  $i$ th joint axis.

**Step - 5:** Assign  $x_i$  axis pointing along the common normal ( $a_i$ ) in the direction from  $z_i$  axis to  $z_{i+1}$  axis. In the case of  $a_i = 0$ ,  $x_i$  is normal to the plane of  $z_i$  and  $z_{i+1}$  axes.

- As seen from figure 3.7, the joints may not necessarily be parallel or intersecting. As a result, the  $z$ -axes are skew lines. There is always one line mutually perpendicular to any two skew lines, called the common normal, which has the shortest

distance between them. We always assign the  $x$ -axis of the local reference frames in the direction of the common normal. Thus, if  $a_i$  represents the common normal between  $z_i$  and  $z_{i+1}$ , the direction  $x_i$  is along  $a_i$ .

- If two joint  $z$ -axes are parallel, there are an infinite number of common normals present. We will pick the common normal that is collinear with the common normal of the previous joint.
- If the  $z$ -axes of two successive joints are intersecting, there is no common normal between them (or it has zero length). We will assign the  $x$ -axis along a line perpendicular to the plane formed by the two axes.

**Step – 6:** The  $y_i$  axis is selected to complete right-hand coordinate system.

**Step – 7:** Assigning of end-effector frame: If the joint  $n$  is revolute, the direction of  $x_n$  is chosen along the direction of  $x_{n-1}$  when  $\theta_n = 0$  and the origin of frame  $\{n\}$  is chosen so that  $d_n = 0$ . If the joint  $n$  is prismatic, the direction of  $x_n$  is chosen so that  $\theta_n = 0$  and the origin of frame  $\{n\}$  is chosen at the intersection of  $x_{n-1}$  with  $z_n$  so that  $d_n = 0$ .

**Step – 8:** The link parameters are determined as mentioned in table 4.

Table 4: Link parameters

Link, $i$	$a_{i-1}$	$\alpha_{i-1}$	$d_i$	$\theta_i$
1				
2				

- $a_{i-1}$  = the distance from  $z_{i-1}$  to  $z_i$  measured along  $x_{i-1}$
- $\alpha_{i-1}$  = the angle between  $z_{i-1}$  to  $z_i$  measured about  $x_{i-1}$
- $d_i$  is the distance from  $x_{i-1}$  to  $x_i$  measured along  $z_i$
- $\theta_i$  is the angle between  $x_{i-1}$  to  $x_i$  measured about  $z_i$

**Step - 9:** Form  ${}^0T_n = {}^0T_1 {}^1T_2 {}^2T_3 \dots {}^{n-1}T_n$ . This gives the position and orientation of the end-effector frame expressed in the base coordinates.

## 5. CASE STUDIES

RR planar manipulator is used to differentiate between D-H classical convention and D-H modified convention.

### 5.1 Case Study for D-H Classical Convention

Formulate the forward kinematic model of two-degree of freedom RR planar manipulator as shown in figure 9. Find the home position of the manipulator. Use classical D-H convention.

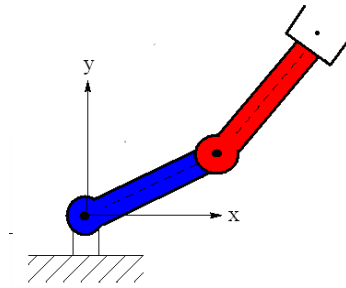


Figure 9: Two-degree of freedom RR Planar manipulator

**Solution:**

The manipulator consists of two joints. The two joints are revolute joints. The scheme of frame assignment of the manipulator is shown in figure 10. For revolute joint,  $d = 0$ .

The manipulator consists of two joints (i.e.  $n = 2$ ). The axis of revolute joint is perpendicular to the paper (figure 10).

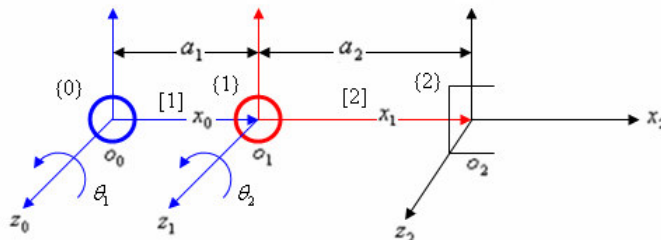


Figure 10: Frame assignment for two-degrees of freedom RR planar manipulator

Step -1: The joints are revolute type. The two links are numbered [1] and [2]. The base frame is {0} and frames for the rest of links are numbered {1} and {2}. The joint axes are labeled as  $z_0$ ,  $z_1$ , and  $z_2$ .

Step -2: The location of frame to the base is arbitrary. The  $x_0$  axis, which is perpendicular to  $z_0$ , is chosen to be parallel to  $x_1$  when the first joint angle variable  $\theta_1 = 0$  in the home position. The  $y_0$  axis is defined by the cross product  $y_0 = z_0 \times x_0$ .

For  $i = 1$ , perform steps 3 to 5.

Step -3: The link 1 has two joint axes,  $z_0$  and  $z_1$ . The  $z_0$  axis is assigned to joint 1. The  $z_1$  axis is assigned to joint 2. There is a common normal between  $z_0$  and  $z_1$  axes. Axis  $z_1$  is parallel to  $z_0$  axis. The origin  $o_1$  is located in any convenient position along  $z_1$  axis as shown in figure 3.12.

Step -4: Axis  $x_1$  is established along the common normal between  $z_0$  and  $z_1$  through  $O_1$ .

Step -5: The  $y_1$  axis is defined by the cross product of  $y_1 = z_1 \times x_1$ .

For  $i = 2$

Step -3: The link 2 has one joint axis,  $z_1$  which is common to link 1 and link 2. The second end of link 2 is rigidly connected to the end-effector. The  $z_2$  axis is set parallel to  $z_1$  axis. Since  $z_1$  and  $z_2$  axes are parallel, the origin  $o_2$  is located in any convenient position along  $z_2$  axis as shown in figure 3.12.

Step -4: The common normal between  $z_1$  and  $z_2$  axes is  $x_2$ . Since the joint 2 is revolute,  $x_2$  axis is chosen in the direction parallel to  $x_1$  axis and passing through the origin  $o_2$ .

Step -5: The  $y_2$  axis is defined by the cross product of  $y_2 = z_2 \times x_2$ .

Step -6: Establish the end-effector frame {2} as shown in figure 3.12.

Step -7: The joint-link parameters are tabulated in table 5.

Table 5: Joint-link parameters of classical convention

Link, $i$	$a_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1	$a_1$	0	0	$\theta_1$
2	$a_1$	0	0	$\theta_2$

Step -8: Form the homogeneous transformation matrices  ${}^{i-1}T_i$  by substituting the above parameters into equation (3.7)

$${}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step- 9: Form  ${}^0T_2 = {}^0T_1 {}^1T_2$ . This gives the position and orientation of the tool frame expressed in base coordinates.

$${}^0T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The home position of the manipulator is corresponding  $\theta_1 = \theta_2 = 0$ . By substituting these values in  ${}^0T_2$ , we get the home position of the manipulator.

$${}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 + a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 5.2 Case Study for D-H Modified Convention

Derive forward kinematics of two-degree of freedom RR planar manipulator as shown in figure 11. Find the home position of the manipulator. Use modified D-H convention.

#### Solution:

The manipulator consists of two joints. The two joints are revolute joints. The scheme of frame assignment of the manipulator is shown in figure 11. For revolute joint,  $d = 0$ . The axis of revolute joint is perpendicular to the paper.

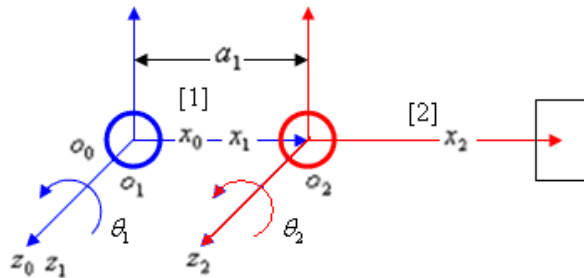


Figure 11: Frame assignment for two-degrees of freedom RR planar manipulator

Step -1: The base frame {0} is assigned to link 0. The base frame {0} is arbitrary. For simplicity chose  $z_0$  along  $z_1$  axis when the first joint variable is zero. Using this convention, we have  $a_0 = 0$  and  $\alpha_0 = 0$ . This also ensures that  $d_1 = 0$  as the joint 1 is revolute.

Step - 2: The manipulator consists of two links. The two links are numbered [1] and [2]. The link frames are numbered as {1} and

{2}. The joint 1 is between link 0 and link 1 and its  $z$  axis is  $z_1$ . The joint 2 is between link 1 and link 2 and its  $z$  axis is  $z_2$ .

For  $i = 1$ , perform steps 3 to 6.

Step – 3: There is a common normal between  $z_1$  and  $z_2$  axes. The origin  $o_1$  of frame {1} is located where the common normal ( $a_1$ ) meets the  $z_1$  axis.

Step – 4: Assign the  $z_1$  axis pointing along the 1st joint axis.

Step – 5: The  $x_1$  axis is pointing along the common normal ( $a_1$ ) in the direction from  $z_1$  axis to  $z_2$  axis and passing through the origin  $o_1$ .

Step – 6: The  $y_1$  axis is selected to complete right-hand coordinate system. For  $i = 2$ , perform steps 3 to 6.

Step – 3: The link 2 consists of only one joint that is  $z_2$  axis and the other end of link 2 is rigidly fixed to the end-effector. There is a common normal between  $z_2$  and  $z_3$  axes (here: the  $z_3$  axis is belonging to the end-effector). The origin  $o_2$  of frame {2} is located where the common normal ( $a_2$ ) meets the  $z_2$  axis.

Step – 4: Assign the  $z_2$  axis pointing along the 2nd joint axis.

Step – 5: The  $x_2$  axis is pointing along the common normal ( $a_2$ ) in the direction from  $z_2$  axis to  $z_3$  axis and passing through the origin  $o_2$ .

Step – 6: The  $y_2$  axis is selected to complete right-hand coordinate system.

Step – 7: Assigning of end-effector frame: The direction of  $x_3$  aligns with  $x_2$  when  $\theta_3 = 0$  and the origin of frame {3} is chosen so that  $d_3 = 0$ .

Step – 8: The link parameters are determined as mentioned in table 6.

Table 6: Link parameters of modified convention

Link, $i$	$a_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1	0	0	0	$\theta_1$
2	$a_1$	0	0	$\theta_2$

Step - 9: Form  ${}^0T_2 = {}^0T_1 {}^1T_2$ . This gives the position and orientation of the end-effector frame expressed in the base coordinates.

$${}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$${}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_{12} \\ s_{12} & c_{12} & 0 & a_1s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Note:** The forward kinematics ends at a frame, whose origin lies on the last joint axis  $z_2$ , therefore,  $a_2$  does not appear in the link parameters.

### 5.3 Difference between Classical and Modified Conventions

The position and orientation of the end-effector frame expressed in the base coordinates obtained by the modified D-H convention are given by:

$${}^0T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_{12} \\ s_{12} & c_{12} & 0 & a_1s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematics ends at a frame, whose origin lies on the last joint axis  $z_2$  as per modified D-H convention, therefore,  $a_2$  does not appear in the link parameters as shown in figure 12. The home position of the manipulator is corresponding  $\theta_1 = \theta_2 = 0$ . By substituting these values in  ${}^0T_2$ , we get the home position of the manipulator.

$${}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

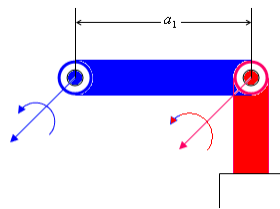


Figure 12: Home position of RR planar manipulator as obtained by modified D-H convention.

The position and orientation of the tool frame derived from the classical D-H convention are given by

$${}^0T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The home position of the manipulator is corresponding  $\theta_1 = \theta_2 = 0$ . By substituting these values in  ${}^0T_2$ , we get the home position of the manipulator as shown in figure 13.

$${}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 + a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

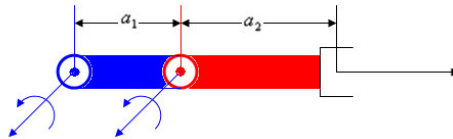


Figure 13: Home position of RR planar manipulator as obtained by classical convention.

## 6. CONCLUSIONS

Two conventions have been established for assigning coordinate frames, each of which allows some freedom in the actual coordinate frame attachment.

D-H parameter	Classical convention	Modified convention
Joint axis	$z_{i-1}$ is for joint $i$	$z_i$ is for joint $i$
Link length ( $a_i$ )	The distance from $o_i$ to the intersection of the $z_{i-1}$ and $x_i$ axes along the $x_i$ axis	The distance from $z_i$ to $z_{i+1}$ measured along $x_i$
Twist angle( $\alpha_i$ )	The angle from the $z_{i-1}$ axis to the $z_i$ axis about the $x_i$ axis	The angle between $z_i$ to $z_{i+1}$ measured about $x_i$
Offset length ( $d_i$ )	The distance from the origin of the $(i-1)$ frame to the intersection of the $z_{i-1}$ axis with the $x_i$ axis along the $z_{i-1}$ axis	The distance from $x_{i-1}$ to $x_i$ measured along $z_i$
Joint angle ( $\theta_i$ )	The angle between the $x_{i-1}$ and $x_i$ axes about the $z_{i-1}$ axis	The angle between $x_{i-1}$ to $x_i$ measured about $z_i$

This paper discusses both the conventions (classical and modified) for the forward kinematics of RR manipulator. One can have choice of using any one method. Most of the universities are having several affiliated technical institutions. In such situations, the question paper is set by the university for the end examinations. It may happen that the answer scripts may consist of either of the conventions. Teachers are requested to correct the

answer scripts as per the convention followed by the student but not the convention that he taught in the class room.

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